VVI

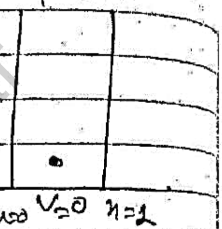
# Application of Schrodinger Theory

Q. Show that the energy of an electron that is confined in an infinite potential well is quantized and hence determine normalized wave function of an electron confined in an infinite potential well.

→ Let us consider a particle (electron) in an infinitely deep potential well defined as

$V = 0$ for $0 < x < L$

$V = \infty$ for $x < 0$ & $x > L$.




@ electron in an infinitely deep potential well.

Now,

general time independent Schrodinger wave equation

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0 \quad \text{--- (i)}$$

For electron inside well, $V=0$

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0 \quad \text{--- (ii)}$$

we have,

$$E = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m} \quad [\because p = \hbar k]$$

or, $k^2 = \frac{2mE}{\hbar^2}$

Now, substituting in eqn (ii)

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0 \quad \text{--- (iii)}$$

Date .. / .. / ..
Page ......

The solution of above differential equation is

$$\psi(x) = A\sin kx + B\cos kx \quad \text{--- (iv)}$$

where, A & B are constants that we have to are calculated using b/n boundary conditions.

Since the particle is confined within the well the probability of finding the particle outside the wall or across the wall = 0, then we have following b.c's,

$\psi(x) = 0$ when $x = 0$ --- (v)

$\psi(x) = 0$ when $x = L$ --- (vi)

Using b.c's (v) in (iv),

$\psi(0) := 0$

or, $A\sin k\times 0 + B\cos k\times 0 = 0$

or, $B\cos 0 = 0$

Since $\cos 0 = 1$

Now, $\therefore B = 0$.

$\psi(0) := 0$

or, $A \sin k \times 0 \to 0 + B \cos k \times 0 = 0$

or, $B \cos 0 = 0$

since $\cos 0 = 1$

Now, $\therefore B = 0.$

$\psi(x) = A \sin kx$ ---- (vii)

Applying b.c's (vi) in (vii)

$\psi(L) = 0$

or, $A \sin kL = 0$

since $A \neq 0$

$\sin kL = 0$

$\sin kL = \sin n\pi$

$\therefore kL = n\pi$

$\therefore k = \frac{n\pi}{L}$

we have, $E = \frac{\hbar^2 k^2}{2m}$

$= \frac{\hbar^2 \left(\frac{n\pi}{L}\right)^2}{2m}$

$= \frac{\frac{h^2}{4\pi^2} \cdot \frac{n^2\pi^2}{L^2}}{2m}$

$= \frac{n^2 h^2}{8mL^2}$

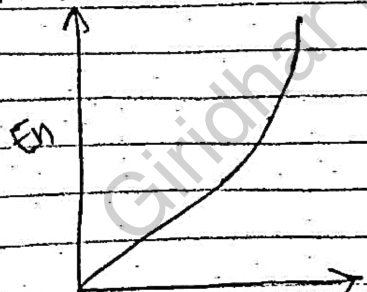
$$\boxed{\therefore E = \frac{n^2 h^2}{8mL^2}} \quad \text{---- (viii)}$$

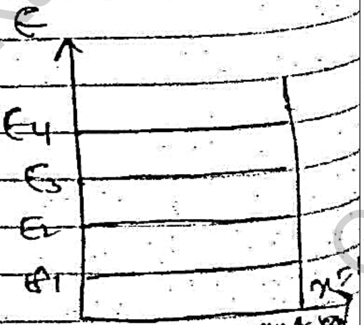
$\therefore$ The energy eigen value of electron in well are,

$E_1 = \frac{h^2}{8mL^2}$, $E_2 = \frac{4h^2}{8mL^2}$

$\therefore E \propto n^2$

$\therefore$ The energy eigen value of electron in infinite potential well is quantized. is proportional to $n^2$.




graph representing

(ii) Diff energy of electron in infinite

Hence normalized wave function

$$f^n \boxed{\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L}\right)x}$$